[This question paper contains 4 printed pages.]

878

May 2013

Your Roll No. ....

B.Sc. (Hons.) / III

C

PHYSICS – PAPER – XIX

(Statistical Physics)

Time : 3 Hours

Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Five** *questions in all.*
Question No. **1** *is compulsory.*

1. Answer any **five** of the following : (2×5)

   (a) Under what conditions Bose-Einstein and Fermi Dirac distributions reduces to classical distribution. Show graphically such variations.

   (b) What are the main constituents of white dwarf stars ? What prevent the gravitational collapse of white dwarf stars ?

   (c) Explain whether the law of equipartition of energy can be applied to the system of classical harmonic oscillators.

   (d) Discuss any one application of Richardson Dushman equation of thermionic emission.

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(e) State Wien's Displacement Law.

(f) Distinguish between the macrostates and microstates.

(g) State Kirchhoff law of thermal radiation and give one application of it.

UNIT – I

2. (a) Derive Sackur- Tetrode equation for the entropy of an ideal gas. Show that the entropy of mixing is zero for a reversible process.

(b) Suppose the number of microstates, $\Omega(U, N)$ of an isolated system is described by

$$\Omega = \frac{U^N}{N!}$$

Assuming N is very large, find entropy $S(U, N)$, internal energy $U(S, N)$ and temperature for such system. (4,3)

3. Consider a paramagnetic solid consisting of N particles each having a magnetic moment μ, that can either be parallel or antiparallel to an external magnetic field B.

(i) Find the total internal energy and the entropy for such a system.

(ii) Discuss the case when the entropy of such a system is maximum.

(iii) Explain how it is possible for such a system to attain negative temperature. (3,2,2)

UNIT – II

4. (a) Derive the expression for pressure exerted by diffuse radiation.

(b) Establish the relation between the volume and temperature for blackbody radiation undergoing adiabatic expansion in a spherical enclosure of perfectly reflecting walls.

(c) Using Planck Radiation formula, derive the Stefan's law. (3,2,2)

5. (a) Explain the principle and working of a three level laser. Discuss its limitations.

(b) Find the ratio of spontaneous to stimulated emission for a two level system in thermal equilibrium. (4,3)

UNIT – III

6. (a) Derive the expression for the number of quantum states in the energy range ε and $\varepsilon + d\varepsilon$ for a massless relativistic particle. (2)

P.T.O.

(b) Derive the expressions of internal energy, specific heat and entropy for the photon gas. (5)

7. (a) Discuss briefly the quantization of rotational motion. (1)

(b) Derive the expression of rotational partition function and calculate the rotational contribution to the specific heat capacity of a diatomic gas. Show that it reduces to the classical value at high temperature. (1,4,1)

UNIT – IV

8. (a) Obtain an expression for thermodynamic probability of a macrostate using Fermi-Dirac statistics. Hence derive the equilibrium distribution function for a Fermion system having fixed number of particles and energy at temperature T. (1,4)

(b) Plot the distribution function as a function of energy for a electron gas at $T = 0$ K and $T > 0$ K. (2)

9. (a) Explain the terms weak and strongly degenerate gas. (2)

(b) Derive the expressions of chemical potential, Pressure and specific heat for a strongly degenerate system of electrons. (5)

(1600)

This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : 6242

Unique Paper Code : 222602 D

Name of the Paper : Statistical Physics [PHHT-620]

Name of the Course : B.Sc. (Hons.) Physics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all.

Question No. 1 is compulsory.

All questions carry equal marks.

Symbols have their usual meanings.

1. Attempt any *five* of the following : $5\times3=15$

   (*a*) Differentiate between a canonical and grand canonical ensemble.

   (*b*) How is the thermodynamic probability of a system different from the conventional probability of an event ? Give the range of the values for both.

   (*c*) Why is it necessary to have population inversion for the existence of negative temperature ?

P.T.O.

(*d*) Give *two* applications of Kirchhoff's law of thermal radiation.

(*e*) The thermodynamic probability of an ideal gas increases from 10^{10} to 10^{100}. Find out the change in entropy.

(*f*) Graphically compare the distribution functions for the three statistics.

(*g*) When is a gas called degenerate ? How is a completely degenerate Fermi gas different from a completely degenerate Bose gas ?

(*h*) State the principle of equipartition of energy. Give *two* examples where it is not valid.

2. (*a*) Derive Saha's ionization formula. Mention its uses. 10

(*b*) Deduce the Boltzmann relation $S = k \ln \Omega$. 5

3. (*a*) What is Gibb's paradox ? 7

(*b*) Deriving Sackur-Tetrode equation, explain how the paradox was resolved ? 8

4. (*a*) What are the basic assumptions of Planck's theory of blackbody radiation ? Derive Planck's radiation formula treating radiation as a collection of oscillators. 10

(b) Using this law, obtain : 5

(i) Wien's displacement law and

(ii) Stefan's law.

5. (a) Show that the electron gas in a white dwarf star of mass $M = 2 \times 10^{30}$ kg and density $\rho = 10^{10}$ kg m^{-3} is highly degenerate and relativistic. The temperature of the star is of the order of 10^6 K. 4

(b) Obtain the mass-radius relationship for such a star. 8

(c) Show that the star cannot exist as a white dwarf star if its mass is greater than the Chandrasekhar mass limit. 3

6. (a) A system of bosons cannot have a positive chemical potential. Explain. 3

(b) For a strongly degenerate Bose gas, find the expressions for the internal energy U, entropy S, specific heat C_v and show that $C_v = 1.5$ S. 8

(c) With the help of graphs, show the variation of the fraction of bosons in the ground state and that in the excited states, with temperature. 4

7\. (*a*) Write the expression for the total number of microstates for a system obeying F-D statistics. Using this, obtain the F-D distribution function. 5

(*b*) Plot the distribution as a function of energy for T = 0 K and T > 0 K. 4

(*c*) Obtain the expression for the Fermi energy in terms of the density of fermions. If the Fermi energy of a given metal is 4.74 eV, find the density of electrons in it. 6

[This question paper contains 4 printed pages.]

5269 Your Roll No. ....

B.Sc. (Hons.) / III D

PHYSICS – PAPER XIX

(Statistical Physics)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** *is compulsory.*
Attempt **one** *question from each of the units.*

1. Answer any **five** of the following :

(a) Calculate the number of ways of arranging three Fermions in four available states.

(b) Show that for adiabatic expansion of black body radiation

$$TV^{1/3} = \text{constant}$$

(c) Derive the relationship between energy density of diffuse radiation and the pressure exerted by it.

(d) Explain the significance of partition function in statistical thermodynamics.

P.T.O.

(e) Can negative temperatures exist in a system having energy levels going to infinity ? Give reasons.

(f) What are the drawbacks of a two-level laser ?

(g) Compare the properties of Fermi Gas and Bose Gas at $T = 0$ K. (2×5)

UNIT – I

2. Obtain the expression for the thermodynamic probability of a system following Maxwell Boltzmann Statistics and hence deduce most probable distribution function in terms of chemical potential and temperature. (2,3,2)

3. (a) State and prove principle of equipartition of energy. (4)

   (b) Discuss its relevance in the study of specific heat of a monoatomic gas and diatomic gas. (3)

UNIT – II

4. (a) Derive Saha's Ionisation formula.

   (b) Give any one of its practical application. (6,1)

5. (a) Derive Planck's law of Black Body radiation. Discuss its limiting cases for large and small wavelengths.

(b) Distinguish between spontaneous and stimulated emission. (5,2)

UNIT – III

6. (a) Derive the expression for the temperature at which Bose Einstein condensation occurs. (4)

(b) Explain the relationship between Bose Einstein condensation and Lambda point transition in Helium. (3)

7. Distinguish between Ortho and Para hydrogen. Explain the relative proportion of Ortho and Para hydrogen at room temperature and absolute zero. (2,5)

UNIT – IV

8. Derive Richardson Dushman equation for thermionic emission of electrons from a metal. How can it be used to calculate the work function of a metal ? (7)

P.T.O.

9. (a) Define Fermi Energy and derive the expression for it. (3)

(b) If the Fermi energy of a metal is 3.15 eV, find the Fermi energy of another metal whose number density is nine times that of the former metal. (2)

(c) Show that the electron gas in a White Dwarf Star is relativistic and completely degenerate. (2)

(*c*) Determine the wavelength corresponding to the maximum emissivity of a black body at a temperature T equal to 3°K and 5000°K. Take $b = 2898$ μmK. In what spectral region will the wavelengths be found ?

(*d*) Explain Bose-Einstein condensation. How does it differ from ordinary condensation ?

(*e*) Derive conditions for a strongly degenerate gas. How does the degeneracy depend upon the temperature, number density and mass of particles ?

(*f*) Derive the expression for the Fermi momentum of a collection of electrons with number density *n*.

(*g*) Calculate the partition function, energy and specific heat for a classical system of N particles and three energy levels $0, \epsilon, 2\epsilon$.

(*h*) The Fermi energy for metal-A is 3.15 eV. Find its value for metal B given that the free electron density in metal B is nine times that in metal A.

2. (*a*) State and derive the law of equipartition of energy. Discuss its relevance and limitations with respect to the specific heat of a diatomic gas.

(*b*) What do you understand by partition function ? Derive expressions for internal energy (U), entropy (S) and specific heat (C_V) in terms of partition function. 10,5

3. (*a*) What is the thermodynamic definition of temperature ? Explain the emergence of negative temperatures in a system of magnetic dipoles with spin half in a magnetic field.

(*b*) How is entropy related to probability ? Derive a relation between them. 11,4

4. (*a*) What are the basic assumptions of Planck's theory of black body radiation ? Derive Planck's law of black body radiation. Under what conditions does this law reduce to Rayleigh Jeans's law and Wien's law ?

(*b*) Deduce the expression for Stefan's constant from Planck's black body radiation formula. 10,5

5. (*a*) What is Bose-Einstein distribution law ? Derive expressions for energy, entropy, specific heat and pressure of strongly degenerate Bose gas.

(*b*) Discuss an example of Bose-Einstein condensation.

(*c*) Calculate the number of ways of arranging four Bosons in seven different states. 10,3,2

P.T.O.

6. (*a*) What is the number of ways in which N_i Fermions can be distributed in g_i states ? Find the average occupation number of Fermions in a state with energy ϵ_i.

(*b*) Plot and explain the variation of distribution function for a Fermi gas.

(*c*) Evaluate the temperature at which there is 1% probability that a state, with energy 0.5 eV above Fermi energy, will be occupied by an electron. 8,4,3

7. (*a*) Show that the matter in white dwarf stars behaves like a strongly degenerate relativistic electron gas. Obtain an expression for mass radius relationship.

(*b*) What is the physical significance of Chandershekhar mass limit ? 12,3

[This question paper contains 4 printed pages.]

14 MAY 2016

Sr. No. of Question Paper : 5789 F Your Roll No..............

Unique Paper Code : 222602

Name of the Paper : Statistical Physics (PHHT–620)

Name of the Course : B.Sc. (Honours) Physics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.
3. Question number 1 is compulsory.
4. Symbols have their usual meaning.

1. Answer any five of the following:

   (a) Describe two applications of Kirchhoff's law of black body radiation.

   (b) Find the number of microstates for a system of three particles and three quantum states, if the system obeys M-B, B-E and F-D statistics.

   (c) A photon gas is an ideal boson gas. Justify this statement by giving reasons.

   (d) The density of conduction electrons in a metal is 0.1 kg/m^3. Calculate the Fermi energy and Fermi temperature for the metal.

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(e) A body at 1600 K emits maximum energy at a wavelength 20,000 A°. If the sun emits maximum energy at wavelength 5500 A° what would be the temperature of the sun ?

(f) Define ensemble. Distinguish between micro canonical, canonical and grand canonical ensembles.

(g) What is Hertzsprung-Russell diagram ? Discuss the position of white dwarf and red giant stars on this diagram.

(h) Can negative temperature be achieved in a system having infinite number of energy levels ? Justify your answer. (5×3)

2. (a) State and derive the principle of equipartition of energy using statistical method and discuss two of its limitations.

(b) Establish the relation $S = k_B \log \Omega$, where Ω is thermodynamics probability. (9,6)

3. (a) What are the basic assumptions of Planck's theory of blackbody radiation ? Derive Planck's radiation formula for a blackbody treating it as a collection of oscillators.

(b) Use Planck's radiation formula to obtain Stefan's constant. (10,5)

4. (a) What is ultraviolet catastrophe ?

(b) For a particular temperature the emissivity and absorption coefficient at wavelength 1000 Å for a body is 8 and 0.5 units respectively. Deduce the emissivity for a black body at the same temperature and wavelength.

(c) Derive Sana's ionization formula and mention two of its important applications.

(3,3,9)

5. (a) What is Bose-Einstein condensation and how is it different from ordinary condensation ? Show that the temperature at which the onset of Bose-Einstein condensation occurs is given by

$$T_c = \frac{h^2}{2\pi mk}\left[\frac{N}{2.612V}\right]^{2/3}$$ where k is Boltzmann's constant.

(b) Obtain the fractions of bosons in the condensed phase (N_{con}/N) and in the excited state (N_{ex}/N) as a function of temperature parameter (T/T_c). Represent them graphically.

(10,5)

6. (a) Obtain the expression for the thermodynamic probability(Ω)of a fermion system and hence deduce its distribution function

$$\langle n(\varepsilon)\rangle = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

(b) Show that the specific heat at constant volume for an ideal Fermi gas at $T \ll T_F$ is given by

$$C_v = \frac{\pi^2}{2} Nk_B\left(\frac{T}{T_F}\right)$$

where T_F is Fermi temperature.

(10,5)

7. (a) Consider a system of N magnetic dipoles in a microcanonical ensemble under a magnetic field B. Enumerate the number of microstates, $\Omega(N,E)$, accessible to the system at energy E and hence

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evaluate the expressions for energy and entropy. Discuss the attainability of negative temperature for such a system with suitable schematic diagrams.

(b) Show that the maximum entropy for this system is given by

$$S = Nk_B \log 2$$

(10,5)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: 5793	B
Unique Paper Code	: 222602	
Name of the Paper	: Statistical Physics	
Name of the Course	: B.Sc. (Hons.) Physics	
Semester	: VI	
Duration	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all, including Question No. 1 which is compulsory.

1. Answer any five of the following :

(a) Explain why the heat capacity at constant volume of a nitrogen molecule is roughly 1.5 Nk at one Kelvin temperature but 2.5 Nk at room temperature. (K is Boltzmann constant).

(b) Discuss and explain the validity criterion of classical statistics in terms of temperature and number density of particles.

(c) Can negative temperature be achieved in a system consisting of harmonic oscillators? Justify your answer.

(d) Compare the thermodynamical properties of ^{4}He atoms and electrons at T = 0 K.

(e) Explain how a metal remains solid in spite of the large Fermi pressure due to electrons.

(f) Show that the Planck's law for blackbody radiation reduces to Rayleigh–Jeans law in the long wavelength region.

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(g) A black body has its cavity of cubical shape. Determine the number of modes of vibration per unit volume in the wavelength range of 4990 – 5010 Å.

(h) Differentiate between degenerate energy level and degenerate gas. 5 x 3

2. (a) Define thermodynamic probability and hence establish the relation between thermodynamic probability and entropy. Identify the constant appearing in the expression.

(b) Consider a gas consist of N distinguishable particles. Every particle has access to two non-degenerate energy levels of energy 0 and ε. The gas occupies the fixed volume and also in thermal equilibrium with a reservoir at temperature T. Calculate the internal energy and entropy of such system. 10, 5

3. (a) Prove that the partition function for an ideal monoatomic gas consisting of N indistinguishable particles is given as:

$$Z = \frac{V^N}{N!}\left(2\pi\frac{mkT}{h^2}\right)^{3N/2}$$

Explain the significance of 1/N! term in the above expression.

(b) Using the above expression, derive the Sackur-Tetrode equation for the entropy. Hence show that entropy is behaving as an extensive parameter. 9, 6

4. (a) Write an expression for thermodynamical probability of a macrostate for a system obeying Bose-Einstein statistics and hence obtain the equilibrium

distribution function for bosons having fixed number of particles and energy at temperature T.

(b) Show that below the condensation temperature (T_C), the specific heat of a ^4He gas is given by :

$$C_V = 1.92\, Nk\, (T/T_C)^{3/2}$$ 8,7

5. (a) Consider a photon gas enclosed in a volume V in equilibrium with temperature T. Show that the average number of photons in this volume V is proportional to T^3.

(b) Starting from distribution function for Fermi Dirac statistics, derive an expression for Fermi momentum of electrons at T=0 K and hence show that the de Broglie wavelength associated with the electrons is:

$$\lambda = 2(\pi/3n)^{1/3}$$

where n is the number density of the electron gas. 8,7

6. (a) What is Hertzsprung-Russell diagram? Discuss the position of white dwarf and red giant stars on this diagram.

(b) Consider the following model of a white dwarf star: a gas sphere consisting of helium of mass $M=10^{30}$ kg at a density of $\rho=10^{10}$ kg m^{-3} and a (central) temperature T is of the order of 10^6 K. Show that electron gas inside the white dwarf star is strongly degenerate and highly relativistic.

(c) Derive the expression of pressure generated by relativistic strongly degenerate electron gas inside the white dwarf stars. 3,4,8

7. (a) State and prove Kirchhoff's law of blackbody radiation.

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(b) Use Planck's law for blackbody radiation obtain Wien's constant. 9

8. (a) Derive an expression for the critical temperature (T_c) of Bosons.

(b) Determine the expression for the internal energy E, possessed by strongly degenerate non-relativistic electrons ($T < T_F$).

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2268A IC

Unique Paper Code : 32221602

Name of the Paper : Statistical Mechanics

Name of the Course : **B.Sc. (H) Physics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Non-programmable Scientific Calculators are Allowed.

SECTION A

Question No. 1 *is compulsory.*

1. Attempt all parts of this question :

(a) Find out the total number of ways of filling 3 particles in two energy groups of 4 cells each

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so that 2 particles are placed in one energy group and one particle in the other. Particles here are (i) identical and distinguishable (ii) identical indistinguishable and (iii) identical indistinguishable and obeying Pauli exclusion principle.

(b) Find out the condensation temperature for He II given that the volume occupied by 1 g mole of the gas is 27.4 cm^3.

(c) Find the Fermi energy of the electrons in silver. Given that atomic weight is 108, density being 10.5 gm/cm^3.

(d) Given a Fermi gas, what is the mean occupation number for a state with energy $3kT/2$ above the Fermi energy?

(e) A cavity of volume V, and at temperature T is filled with radiation. If the radiation pressure is 1 atm, what is its temperature? (1 atm = 1.01×10^5 N/m^2)

(f) Consider a system of three distinguishable particles with particle energies 0, σ, 2ε, 3ε ..... Let the total energy of the system be 3ε. Enumerate all macrosates and microstates of the system. What is the probability that at least one particle has energy ε. (3×5,4)

SECTION B

Answer any **four** *questions from the following.*

Attempt any **two** *parts from each question.*

Each part carries 7 marks.

2. (a) Derive Sackur-Tetrode equation for the entropy of an ideal monoatomic gas. How does it resolve the Gibbs paradox?

(b) A system of N weakly interacting, distinct particles is such that each particle can be visualised as a three-dimensional harmonic oscillator with energy

$$E = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + \frac{1}{2}m\omega^2\left(x^2 + y^2 + z^2\right).$$

Assuming that the system is in thermal equilibrium at temperature T, compute (a) the mean square speed v^2 and (b) mean square displacement r^2 of a particle, where $r^2 = x^2 + y^2 + z^2$.

(c) For a system of N particles, distributed in 2 non-degenerate energy states $-\in$ and $\in$, find the internal energy and entropy at temperature T? Show graphically the variation of internal energy and entropy with temperature.

3. (a) Show that the single particle partition function for an ideal monatomic gas enclosed in volume V and at temperature T is

$$Z = \left(\frac{2\pi mkT}{h^2}\right)^{3/2}.$$

Find the average energy and pressure for a system of N such distinguishable particles.

(b) For a system of N particles distributed in 2 energy states and degeneracies given by

$\epsilon_1 = 0,\ g_1 = 1$ and $\epsilon_2 = 2\epsilon,\ g_2 = 4$.

Find the ratio of particles in the ground and excited state at a temperature T. Also calculate the internal energy of such a system at temperature T.

(c) Consider a system of N particles accommodated in non-degenerate states of energy $0, \epsilon, 2\epsilon, \ldots$. Calculate thermodynamic probability for the system for (i) $E = \epsilon$ and (ii) $E = 2\epsilon$, where E is the total energy of the system. What is the temperature of the system as E changes from e to 2ϵ?

4. (a) Derive Saha's ionisation formula for the degree of ionisation of gas in a star.

(b) Using Planck's law of black body radiation

$$u_\nu \, d\nu = \frac{8\pi h\, \nu^3 d\nu}{c^3\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

derive an expression for the total energy density of radiation. Compare the specific heat of black

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body radiation in a cavity of volume V at temperature T, with the specific heat of a monoatomic gas of N particles at same temperature and volume.

$$\left(\text{Given } \int_0^{\infty} \frac{x^3}{e^x - 1} = \frac{\pi^4}{90}\right).$$

(c) A planet of radius R_P is in a circular orbit around a star of radius R_0. The obital radius is r. The star can be assumed to be emitting radiation at a constant rate as a black body. Assuming the planet too to be a black-body, which both absorbs the radiation from the star and emits it into the space, find its equilibrium temperature as a function of star temperature T_0. The planet can be approximated to be a disk of radius R_P.

5\. (a) Give Bose's derivation of Planck's black body radiation formula.

(b) A system of five non-interacting spin-less Bosons in equilibrium is such that each particle can be in the ground state with energy $E_0 = \varepsilon$ or in an excited state with energy $E_1 = 3\varepsilon$. The ground state is non-degenerate and the excited state is four-fold degenerate. If the total energy of the system is $E = 11\varepsilon$, what is the entropy of the system?

(c) Show that a system of bosons undergoes BE condensation when the number density of bosons exceeds $\xi\left(\frac{3}{2}\right)(2\pi mkT)^{\frac{3}{2}}/h^3$.

6. (a) What are the contents of a white dwarf star? Show that electrons in a white dwarf star behave like a strongly degenerate relativistic gas.

(Parameters of a white dwarf star: $M = 2 \times 10^{30}$ kg, $\rho = 10^{10}$ kg/m^3 and $T = 10^7$ K.)

(b) For a completely degenerate Fermi Dirac gas of N molecules the density of states is given by :

$$g(\epsilon)d\epsilon = \alpha g_s V \epsilon^n$$

where a, and n are constants, g_s is spin degeneracy and V is the volume. Calculate the Fermi energy and total energy of the system at zero Kelvin temperature.

(c) Calculate the Fermi momentum for 4.2×10^{21} electrons in a box of volume 1 cm^3. How will the Fermi momentum and Fermi temperature change if number of electrons in the same volume is doubled.

Constants

$k = 1.38 \times 10^{-23}$ J/K

$m = 9.11 \times 10^{-31}$ Kg

$b = 2.898 \times 10^{-3}$ m-K

$\sigma = 5.67 \times 10^{-8}$ $J/m^2/s\text{-}K^4$

$h = 6.626 \times 10^{-34}$ J-s

Avogadro No. = 6.023×10^{23} particles